Coulomb's Law :

Charles Augustin de coulomb, who investigated the repulsion between small balls that he charged by rubbing process. To measure the force between the balls, he used a delicate torsion balance similar to the torsion balance later used by Henry Cavendish to measure gravitational forces.

His experimental results are summarized in Coulomb's law:

" The magnitude of the electric force that a particle exerts on another particle is

(i) directly proportional to the product of their charges;

(ii) Inversely proportional to the square of the distance between them.

(iii) The direction of force is along the line joining the particles

Mathematically, the electric force F that a particle of charge q_1 exerts on particle of charge q_2 at a distance r is given by

$$F = K \frac{q_1 q_2}{r^2}$$

where K is proportionality constant.

Note 1

The electric force that the particle of charge q_1 exert on the particle of charge q_2 has the same magnitude as force exerted by q_2 on q_1.

Note 2

(i) Like charges repel each other.

(ii) Unlike charges attract each other.

S.I unit value of K is constant

$$K = 8.99 \times 10^9 \ \frac{N\text{-}m^2}{C^2} \simeq 9 \times 10^9 \ \frac{N\text{-}m^2}{C^2} \ (m/F)$$

Mathematically

$$K = \frac{1}{4\pi\epsilon_0}$$

The quantity ϵ_0 ("epsilon nought") is called the electric constant or the permittivity constant.

* The value of $\epsilon_0 = 8.85 \times 10^{-12} \ \frac{C^2}{N\text{-}m^2} \ (F/m)$

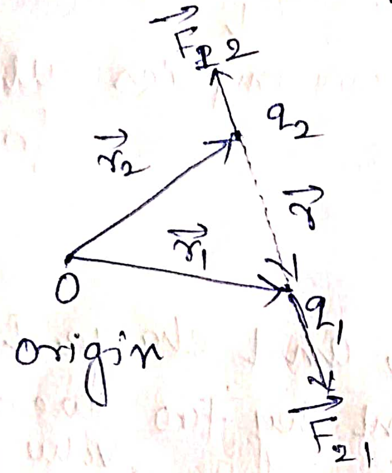
In terms of permittivity constant coulomb's law can be written as

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Coulomb's law in vector notation.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

Let the location of charge q_1 is $\vec{r_1}$ and q_2 is $\vec{r_2}$



$\vec{F}_{12}$ is force on q_2 due to q_1

$$\vec{r} = \vec{r_2} - \vec{r_1}$$

$$\therefore \vec{F}_{12} = K \frac{q_1 q_2}{|\vec{r_2} - \vec{r_1}|^2} \hat{r}$$

$\hat{r}$ ↳ unit vector

$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}$$

$$\vec{F}_{12} = K \frac{q_1 q_2}{|\vec{r_2} - \vec{r_1}|^2} \frac{(\vec{r_2} + \vec{r_1})}{|\vec{r_2} - \vec{r_1}|}$$

$$= \frac{K q_1 q_2}{|\vec{r_2} - \vec{r_1}|^3} (\vec{r_2} - \vec{r_1})$$

$\vec{F}_{21}$ is force on q_1 due to q_2

$$\vec{F}_{21} = |F_{12}| (-\hat{r})$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

Note :

Sign of charges must be taken in account so that

(i) For like charges $q_1 q_2 > 0$ implies repulsive force

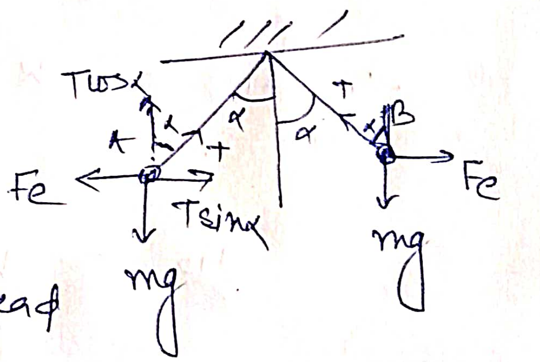
(ii) for unlike charges $q_1 q_2 < 0$ $\downarrow$ implies attractive force.

Problem 1 ;

Two point charges of equal mass m and charge q are suspended at a common point by two threads of negligible mass and length l. Find the value of inclination of mass 'α'? If the angle of inclination of two masses are small and same.

Solution:

Consider system of charges as shown in figures;



where Fe is Coulomb force

T is the tension in each thread

mg is weight of each charge

$$T\sin\alpha = F_e \quad -(1)$$

$$T\cos\alpha = mg \quad -(2)$$

On dividing eqn (1) and eqn (2)

we get $\tan\alpha = \frac{F_e}{mg}$

$$\tan\alpha = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}\frac{1}{mg}$$

where r is distance between two particles

$r = 2l\sin\alpha$

for small α $\quad \sin\alpha \simeq \tan\alpha \simeq \alpha$

$$\therefore \alpha = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4l^2\alpha^2}\frac{1}{mg} \Rightarrow \alpha^3 = \frac{q^2}{16\pi\epsilon_0 mgl^2}$$